

# The New Prime theorem (22)

Hardy-Littlewood conjecture B:  $P, P+k$

Chun-Xuan Jiang

jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove Hardy-Littlewood conjecture B:  $P, P+k$  [4].

**Theorem.** We define prime equation

$$P_1 = P + k. \quad (1)$$

For every even  $k$  there are infinitely many primes  $P$  such that  $P_1$  is a prime.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1-\chi(P)], \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$q+k \equiv 0 \pmod{P}, \quad q=1, \dots, P-1. \quad (3)$$

If  $P|k$  then  $\chi(P)=0$ ,  $\chi(P)=1$  otherwise.

Substituting it into (2) we have

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P|k} \frac{P-1}{P-2} \neq 0. \quad (4)$$

We prove that there are infinitely many primes  $P$  such that  $P_1$  is a prime.

We have asymptotic formula

$$\pi_2(N, 2) = \left| \{P_1 < N : P+k = \text{prime}\} \right| \sim \frac{J_2(\omega)}{\phi^2(\omega)} \frac{N}{\log^2 N}, \quad (5)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

**Remark.** The prime number theory is basically to count the Jiang function  $J_{n+1}(\omega)$  and Jiang

prime  $k$ -tuple singular series  $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_p \left(1 - \frac{1+\chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  [1,2], which can count

the number of prime number. The prime distribution is not random. But Hardy prime  $k$ -tuple singular series

$\sigma(H) = \prod_p \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  is false [3-8], which can not count the number of prime numbers.

## References

- [1] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002, MR2004c:11001, (<http://www.i-b-r.org/docs/jiang.pdf>) (<http://www.wbabin.net/math/xuan13.pdf>)(<http://vixra.org/numth/>).
- [2] Chun-Xuan Jiang, Jiang's function  $J_{n+1}(\omega)$  in prime distribution.(<http://www.wbabin.net/math/xuan2.pdf>.) (<http://wbabin.net/xuan.htm#chun-xuan>)(<http://vixra.org/numth/>)
- [3] Chun-Xuan Jiang, The Hardy-Littlewood prime  $k$ -tuple conjecture is false.(<http://wbabin.net/xuan.htm#chun-xuan>)(<http://vixra.org/numth/>)
- [4] G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. Acta Math., 44(1923)1-70.
- [5] W. Narkiewicz, The development of prime number theory. From Euclid to Hardy and Littlewood. Springer-Verlag, New York, NY. 2000, 333-353.这是当代素数理论水平.
- [6] B. Green and T. Tao, Linear equations in primes. To appear, Ann. Math.
- [7] D. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples I. Ann. Math., 170(2009) 819-862.
- [8] T. Tao. Recent progress in additive prime number theory, preprint. 2009. <http://terrytao.files.wordpress.com/2009/08/prime-number-theory-1.pdf>
- Szemerédi's theorem does not directly to the primes, because it can not count the number of primes. It is unusable. Cramér's random model can not prove prime problems. It is incorrect. The probability of  $1/\log N$  of being prime is false. Assuming that the events " $P$  is prime", " $P+2$  is prime" and " $P+4$  is prime" are independent, we conclude that  $P$ ,  $P+2$ ,  $P+4$  are simultaneously prime with probability about  $1/\log^3 N$ . There are about  $N/\log^3 N$  primes less than  $N$ . Letting  $N \rightarrow \infty$  we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuple conjecture, but can not prove and count any prime problems[6].
- 本文是 1923 年由 Hardy and Littlewood 提出来到今天仍没解决只能算一个猜想, 利用 Jiang function 给出严格证明, 它是一个最简单素数定理。