Letters to the Editor.

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Dutch Pendulum Observations in Submarines.

DR. F. A. VENING MEINESZ, commissioned by the Dutch Geodetic Committee to make pendulum observations on board the Submarine K II of the Royal Dutch Navy during the voyage from Holland to Java (see NATURE of September 15, p. 393), has sent particulars of his observations from Gibraltar, Tunis, and Alexandria.

The beginning of the voyage was extremely disappointing because of the bad weather. For the first five days the sea was continually very rough. The rolling of the ship amounted to 30° to each side, and the pitching to 8 metres; the nights had to be spent strapped to the berths. It was a very rough experience for the first stay on board a seagoing vessel.

After passing Portland Bill in the English Channel, an attempt was made to take observations. Submerged to a depth of 20 metres, the rolling still amounted to ¾° to each side, which made observations impracticable. At length, off the Portuguese coast, the weather cleared and it became calmer, but the long swell continued. On September 24 an inquiry was made again into the movements of the submerged ship. The greatest angle of inclination caused by the pitching amounted at the sea-surface to 1°, the rolling to 6° to each side. At a depth of 30 metres, and while the vessel was going in the direction of the swell, the inclination caused by the pitching was at most ¾°, which by the use of the horizontal rudder could be reduced to less than ¼°; but as the rolling was still 1¼° to each side, observations were practically impossible.

Notwithstanding the considerable rolling of the ship, the amplitudes of the pendulums appeared to vary fairly regularly. The principal impediment was the circumstance that the rays from the electric lamp, reflected by the mirrors of the pendulums, went beyond the edge of the film. The actual trouble was therefore of an incidental nature. This induced Dr. Vening Meinesz to devise an arrangement for suspending the whole apparatus from a horizontal axis to be placed lengthwise in the ship in order to neutralise the rolling. He supposed that it would be possible to get this constructed at the workshops of the Royal Navy at Gibraltar.

On September 26, between Cape St. Vincent and Cadiz, the sea was very smooth, and for the first time observations were crowned with success, as at a depth of 25 metres the movements were very small. The first observation was made in a place where the sea was 110 metres deep, the second where it was 480 metres deep. During the second observation the direction of the course was taken successively W.E. and E.W., to test the effect of the speed of the ship on the intensity of gravity, first mentioned by Eötvös.

On the afternoon of September 28, Gibraltar was reached, and immediately Dr. Vening Meinesz took steps for the construction of the suspension apparatus. All the assistance desired was kindly given by the British authorities. The time being very limited, it was necessary to carry on the work day and night without intermission.

During the stay at Gibraltar the observations were worked out, and they proved to be very successful. The discrepancies of the observations showed the accuracy to be greater than was expected from the preliminary observations at the Helder. The effect of the speed of the ship was clearly indicated by the diagrams; the speed could even be derived from these with a difference of but ½ mile from the true value.

On October 3, a few hours before leaving Gibraltar, the suspension apparatus was fitted up on board the submarine. I am glad to express thanks to the British authorities at Gibraltar, who so readily contributed to the realisation of Dr. Vening Meinesz's project.

During the passage between Gibraltar and Tunis, the arrangement proved to be satisfactory in every respect. Although the rolling amounted to 2° to each side, observations were easily practicable. A stay at Tunis, where the submarine arrived on October 7, was again used by Dr. Vening Meinesz for the preliminary computation of his observations. One of these gave the value of g for a sea-depth of 2500 metres with a difference of only 0·003 cm. sec.^{-2} from the theoretical value, which indicates complete isostasy.

Tunis was left on October 13, and Alexandria was reached on October 18; the sea being generally very smooth, observations were made without any difficulty. The Eötvös effect was tested again; the deduced speed of the ship differed only 0·3 mile from the true value.

It appears from the diagrams that the accuracy of the deduced period of oscillation in favourable circumstances may be about 1/1,000,000, and that in a rough sea there is little fear of the divergences exceeding 1/100,000. We must wait, however, for the complete computations before a positive statement will be possible.

It should also be mentioned that the rate of the chronometer was controlled by using the rhythmic time-signals of the Eiffel Tower.

On October 31 the squadron, consisting of the mother ship *Pelikaan* and the three submarines, left Suez; it will touch at the ports of Aden, Colombo, and Sabang, and arrive at Batavia about the middle of December. Dr. Vening Meinesz will carry out observations in the Red Sea and the Indian Ocean, and will ultimately determine, with the invar pendulums, the intensity of gravity at a few stations in Java.

From the results already obtained it may be concluded that, by the method of Dr. Vening Meinesz, investigations of the intensity of gravity by pendulum observations can be realised on the parts of the earth covered by the ocean with almost the same accuracy as on continents and islands. For the study of isostasy, and of Wegener's hypothesis of floating continents, observations in submarines, especially between the coast and the deep sea, will be of the greatest value.

J. J. A. MULLER.

Zeist, November 7.

The True Relation of Einstein's to Newton's Equations of Motion.

THE equations of a space-time geodesic or Einstein's general equations of motion of a free particle are, in usual symbols,

$$\frac{d^2x_\iota}{ds^2}+\left\{\begin{matrix}\alpha\beta\\ \iota\end{matrix}\right\}\frac{dx^\alpha}{ds}\frac{dx_\beta}{ds}=0,\ \iota=1,\ 2,\ 3,\ 4. \quad . \quad (1)$$

In order to show their relation to Newton's equations of motion, which may be written

$$\frac{d^2\xi_i}{dt^2}=\frac{\partial\Omega}{\partial\xi_i},\ i=1,\ 2,\ 3, \quad . \quad . \quad . \quad (N)$$

Einstein considers the special case of slow motion in a weak gravitation field, *i.e.* such that the metrical tensor components $g_{\iota\kappa}$ differ but little from their Galileian values. Then, neglecting squares, etc., of these small differences and also their derivatives with respect to x_4 (quasi-stationary field), Einstein easily obtains the Newtonian equations as a first approximation, with $\Omega = -\frac{1}{2}c^2 g_{44}$ as the classical potential of the gravitation field. This treatment of the question is repeated, so far as I know, by all exponents of Einstein's theory.

Now, as has recently occurred to me, the true relation of Einstein's equations to those of Newton is of a much more intimate nature, and remains valid, no matter how strong the field and how much space deviates from Euclidean behaviour.

In fact, the frame most natural to adopt for an interpretation of the complicated equations of motion (1) of a particle being clearly its own *rest-system*, let x_1, x_2, x_3 be the space-coordinates of the particle in such a system (the latter, of course, to play its part during an infinitesimal time and to be replaced successively by others and others). Moreover, let for convenience the origin of x_1, etc., be taken at the particle itself. Then, at any instant, $\dot{x}_i = dx_i/ds = 0 (i = 1, 2, 3)$, and equations (1) will reduce to $ds^2 = g_{44}dx_4^2$ and the three equations

$$\frac{d}{dt}\left(\frac{1}{\sqrt{g_{44}}}\frac{dx_i}{dt}\right) = -\frac{c^2}{\sqrt{g_{44}}}\left\{ {44 \atop i} \right\}, \quad . \quad . \quad (2)$$

where $dt = dx_4/c$, the fourth equation being already utilised. Now, with i, k reserved for 1, 2, 3,

$$\left\{ {44 \atop i} \right\} = g^{ik}\left(\frac{\partial g_{4k}}{\partial x_4} - \frac{1}{2}\frac{\partial g_{44}}{\partial x_k}\right) + \frac{1}{2}g^{i4}\frac{\partial g_{44}}{\partial x_4}.$$

The coordinates can always be chosen so as to make $g^{41} = g^{42} = g^{43} = 0$. This means a frame not spinning relatively to the stars. In these coordinates then, or in such a rest-platform of the particle,

$$\left\{ {44 \atop i} \right\} = -\frac{1}{2}g^{ik}\frac{\partial g_{44}}{\partial x_k},$$

and since the x_i can now always be measured along the principal axes of the operator or matrix g^{ik} (when also $g^{ii} = 1/g_{ii}$), we have

$$\left\{ {44 \atop i} \right\} = -\frac{1}{2g_{ii}} \cdot \frac{\partial g_{44}}{\partial x_i},$$

no more to be summed over i, of course. These values substituted in (2) give, with $g_{ii} = -a_{ii}$, and since $\dot{x}_i = dx_i/dt = 0$,

$$\frac{d^2(\sqrt{a_{ii}}x_i)}{dt^2} = -\frac{c^2}{2}\frac{\partial g_{44}}{\sqrt{a_{ii}}\partial x_i}. \quad . \quad . \quad (3)$$

Now, the space-line element of our platform being

$$dl^2 = a_{11}dx_1^2 + a_{22}dx_2^2 + a_{33}dx_3^2,$$

$\sqrt{a_{11}}dx_1$, etc., are the length elements $d\xi_1$, etc., measured along the axes precisely as in (*N*), and the right-hand member of (3) expresses the gradient of $\Omega = -\frac{1}{2}c^2 g_{44} +$ const. With a proper choice of the constant, $g_{44} = 1 - 2\Omega/c^2$.

We thus see that, *in the rest-system of the free particle, the general relativistic equations* (1) *become identical with the Newtonian equations of motion, rigorously, i.e.* whether the gravitation field is weak or not ($2\Omega/c^2$ a small fraction of unity or not), and no matter how strongly the platform-space differs from a homaloidal or Euclidean space.

This simple investigation is here given not merely because it seems to put the general equations (1) into an interesting and familiar light, but also because it vindicates the rights of the Newtonian equations of motion.

LUDWIK SILBERSTEIN.

129 Seneca Parkway, Rochester, N.Y.,
September 19.

The Influence of Barometric Pressure on the Specific Gravity of the Surface Water in Indian Seas.

IT has for many years been recognised that any alteration in barometric pressure over a wide expanse of water produces concomitant changes in the surface level, and Prof. J. W. Gregory (*Scottish Geographical Magazine*, 1909, vol. xxv. p. 316), when discussing the level of the sea, pointed out that "the sea in an area beneath high air pressure has its surface pushed downwards and the displaced water rises in the adjacent areas." Since the waves of increased barometric pressure occur at approximately the same time of day in each degree of longitude, it follows that each succeeding elevation and depression of the surface level of the sea travels across the ocean like a wave from east to west. In the region of India the barometric pressure normally exhibits in every twenty-four hours a double rise and fall with maxima at approximately 9.45 A.M. and 10.30 P.M. and minima at 3.30 A.M. and 4.30 P.M.

Investigations of the specific gravity (σ_0) of the surface water of Indian seas have revealed a daily double oscillation that occurs simultaneously with, and must, I think, be due to, the alterations of barometric pressure. This oscillation of specific gravity is, however, only clearly seen in the open sea, because in inshore waters it is obscured by other changes due to tidal flow, etc. During the voyage from Bombay to Port Blair, Andaman Islands, in October 1921, a four-hourly record of the specific gravity of the surface water and the barometric pressure was carefully kept, and the results obtained are shown in Fig. 1. This shows very clearly the way in which, as the barometric pressure falls, the specific gravity of the surface water rises, and vice versa, the two curves alternating with one another.

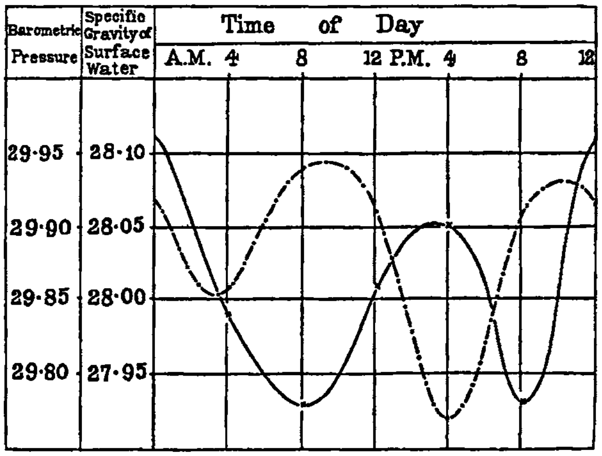

FIG. 1.—Average specific gravity of the surface water and simultaneous barometric pressures during a voyage from Bombay to the Andaman Islands in October 1921.

The continuous line shows the specific gravity, and the dotted line the barometric pressure, in each of the three figures.

A variation in the specific gravity of the surface water such as this might be due to (*a*) lateral horizontal movements of masses of water, or (*b*) an upwelling of water from a deeper level. If the latter cause is the true one, then the effect of changes in barometric pressure should be found to depend on the relative specific gravity of the surface water and of water immediately underlying the surface layer. In October, following on the effects of the south-west monsoon, the upper-level water will be diluted and have a lower specific gravity than that immediately below,