

Can the Lorentz - Fitzgerald Contraction Hypothesis Be Real?

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Abstract

We have shown that choosing a different set of postulates enable us to cancel the Lorentz Transformation (LT) from the main body of special relativity theory (SRT). Hence, this approach excludes the role of the relativistic length contraction, and as a result, the Lorentz-Fitzgerald contraction hypothesis is only a consequence of using the Lorentz's real force (LF). We use the well-known examples to confirm that the relativistic length contraction is not required in relativistic electrodynamics.

Key words: Lorentz force, Lorentz-Fitzgerald contraction, Relativistic length contraction

1 – Introduction

Lorentz deduced LT to solve the following problem: In passing from the absolute rest frame k (The Ether frame) to another inertial frame k' moving with constant velocity u with respect to k , Maxwell's equations change form under Galilean transformations, acquiring additional first and second degree terms in $\frac{u}{c}$. Lorentz noticed that the term at first order term in $\frac{u}{c}$, could easily eliminate by the following alteration

($t' = t \Rightarrow t' = t - \frac{u}{c}x$). But, Lorentz could not explain the null result of the Michelson – Morely experiment without using the contraction hypothesis and at the same time to get rid of the second- order term in $\frac{u}{c}$.

He accepted the hypothesis of Fitzgerald that moving bodies contract i.e., the equations ($x' = \gamma(x - ut)$, $y' = y$, $z' = z$), are real for moving electromagnetic bodies. Lorentz did not prove the real contraction of his transformation equation. The explanation of the Lorentz - Fitzgerald contraction was that because of an interaction with the ether, all bodies contracted in the direction of their motion relative to the ether by a factor $\gamma = 1/\sqrt{1 - u^2/c^2}$.

This hypothesis is important, especially with regard to many calculations in classical electrodynamics. In his paper Einstein [1], derived LT to justify his two postulates. He claimed that the postulates are absolutely real, base on the assumption that LT and its kinematical effects are general laws of Nature.

Einstein showed that the Lorentz - Fitzgerald contraction, which had been introduced previously into classical electrodynamics, is a simple consequence of LT. So after denying the ether Einstein, accepted the contraction hypothesis as the kinematical length contraction of a moving object. According to the Lorentz - Fitzgerald contraction the charge density contracted in the direction of the motion as

$$\rho = \rho_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (1)$$

In Eq. (1), the quantity ρ_0 is the charge density at rest. The methods for deriving Eq. (1) in relativistic electrodynamics textbooks are based on LT and the relativistic length contraction. However, the length of a moving body has never been compared with the length of a body at rest and the hypothesis of "length contraction" in relativistic electrodynamics has been the subject of considerable controversy and re-interpretation [2]. Moreover, to this day it remains a controversial aspect of Einstein's theory [3].

In the papers [4, 5], I suggest another way to account for the kinematics effects in relativistic electrodynamics. This method dose not uses LT for a charged particle and Maxwell fields; instead, it includes LF within the main body of SRT, i.e;

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (2)$$

and applies the relativity principle(that the laws of Physics have the same formulation relative to any inertial system) rather than the special relativity principle(that the laws of physics are invariant under LT). Using only the LF and the relativity principle, instead of Maxwell's equations and the LT, we derived the fundamental relativistic equations. In this paper we clarify some examples in relativistic electrodynamics, which were relativistic electrodynamics textbooks using the relativistic length contraction can be studied according to our method. Thus in the examples contained in this paper, we show that they may be studied without the use of the famous hypotheses of contraction.

2 - Relativistic Transformation Relations without SRT or LT:

The modern physics have in common the relativistic postulate, due to Einstein. An equation which relates physical quantities measured from different reference frames represents a relativistic transformation equation. Starting from Einstein's approach (At first kinematics is involved with length, time and light speed c and then dynamics added to them), one had kinematical effects as time dilation- length contraction.

As demonstrated in paper [4, 5] the relativistic transformation relations are produced without LT and its kinematical effect. We derive the relativistic transformation relations for force, velocity, momentum-energy, and electromagnetic field transformation:

$$F'_x = F_x - \frac{u}{c^2} F_y v_y - \frac{u}{c^2} F_z v_z, F'_y = \frac{F_y}{\gamma \left(1 - \frac{uv_x}{c^2}\right)}, F'_z = \frac{F_z}{\gamma \left(1 - \frac{uv_x}{c^2}\right)} \quad (3)$$

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}, \quad v'_y = \frac{v_y}{\gamma \left(1 - \frac{uv_x}{c^2}\right)}, \quad v'_z = \frac{v_z}{\gamma \left(1 - \frac{uv_x}{c^2}\right)} \quad (4)$$

$$P'_x = \gamma \left(P_x - \frac{u}{c^2} \varepsilon \right), \quad \varepsilon' = \gamma \left(\varepsilon - u P_x \right), \quad P'_y = P_y, \quad P'_z = P_z \quad (5)$$

As well as

$$\begin{aligned} E'_x &= E_x & E'_y &= \gamma (E_y - u B_z) & E'_z &= \gamma (E_z + u B_y) \\ B'_x &= B_x & B'_y &= \gamma \left(B_y + \frac{u}{c^2} E_z \right) & B'_z &= \gamma \left(B_z - \frac{u}{c^2} E_y \right) \end{aligned} \quad (6)$$

Where $\gamma = 1 / \left(1 - u^2/c^2\right)^{\frac{1}{2}}$.

By this approach we get also the Lorentz contracted charge density, i.e.;

$$\rho = \rho_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (7)$$

This was done without using the hypothesis proposed by Lorentz – Fitzgerald or relativistic length contraction [5]. Eq.(7) was a result of our method, while Eq.(1) was an ad-hoc and theoretically unproved result of Lorentz – Fitzgerald contraction.

Example 1:

Let us take a cylindrical charged beam of radius (a) in which the charge density is ρ (line charge density $\lambda = \pi a^2 \rho$), wherein all charged particles travel in frame s with velocity v along its length. We will try to calculate the force that is produced by the charges on a particle of charge q on the surface of the beam, which moves with the same velocity as the charges. The electric field and the magnetic field produced by the moving beam of charges at the location of q are:

$$E = \frac{\lambda}{2\pi a \epsilon_0} = \frac{\rho a}{2\epsilon_0} \quad (a), \quad B = \frac{v \rho a}{2\epsilon_0 c^2} \quad (b) \quad (8)$$

Hence, the Lorentz force on q is

$$F = \frac{q a \rho}{2\epsilon_0} \left(1 - \frac{v^2}{c^2}\right) \quad (9)$$

Now, according to the relativity principle, the transformed force in frame s' is:

$$F' = \frac{q a \rho'}{2\epsilon_0} \left(1 - \frac{v'^2}{c^2}\right) \quad (10)$$

Where $c^2 = 1/\epsilon_0 \mu_0$

On the other hand, if we take the relativistic transformation equation for the perpendicular component of the force, Eq.(3), i.e.;

$$F' = \frac{F}{\gamma \left(1 - \frac{uv_x}{c^2}\right)} \quad (11)$$

and substituting (9) into (11), we obtain for the force acting on q in the frame s'

$$F' = \frac{q \rho a \left(1 - \frac{v^2}{c^2}\right)}{2\epsilon_0 \gamma \left(1 - \frac{uv_x}{c^2}\right)} \quad (12)$$

Now, using the following relation [5]:

$$\frac{1}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{\left(1-\frac{uv_x}{c^2}\right)}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}} \quad (13)$$

in Eq.(12), we get

$$F' = \frac{q\rho a}{2\epsilon_0} \left(1-\frac{v'^2}{c^2}\right)^{1/2} \left(1-\frac{v^2}{c^2}\right)^{1/2}$$

The last relation is equivalent to the following relation

$$F' = \frac{q\rho a \left(1-\frac{v^2}{c^2}\right)^{1/2}}{2\epsilon_0 \left(1-\frac{v'^2}{c^2}\right)^{1/2}} \left(1-\frac{v'^2}{c^2}\right) \quad (14)$$

Now by comparing Eq.(14) with the Eq.(10), we have

$$\rho' = \frac{\rho \left(1-\frac{v^2}{c^2}\right)^{1/2}}{\left(1-\frac{v'^2}{c^2}\right)^{1/2}} \quad (15)$$

The relation (15) leads to the following relations [5]:

$$\rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (a), \quad \rho' = \frac{\rho_0}{\sqrt{1-\frac{v'^2}{c^2}}} \quad (b) \quad (16)$$

In classical electrodynamics one postulated the Lorentz - Fitzgerald contraction to derive Eqs.(16), while in relativistic electrodynamics one uses the relativistic length contraction for the same equations. We have deduced them in this article without using the relativistic length contraction and the Lorentz - Fitzgerald contraction is now a result but not a postulate.

Example 2:

Derivation of the electric field intensity \mathbf{E} and the magnetic induction \mathbf{B} generated by a point charge moving with constant velocity \mathbf{v} was first obtained by O.Heaviside.

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^3} \frac{\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{u^2}{c^2} \sin^2 \theta\right)^{3/2}} \mathbf{r} \quad (17a)$$

$$\mathbf{B} = \frac{\mathbf{v} \times \mathbf{E}}{c^2} \quad (17b)$$

Where \mathbf{r} is the distance between the point where the measurement of \mathbf{E} and \mathbf{B} take place and the charge, θ representing the angle between \mathbf{r} and velocity of point charge \mathbf{v} .

New methods [6, 7, and 8] have been appeared after the emergence of special relativity theory. All these new methods depend on using of LT and on other relativistic transformation relations. But an interesting attempt of "derivation of Eqs. (17) from classical electromagnetism" shown recently by Hajra [9].

In this paper we follow our method which was presented in [4, 5] to derive Eqs (17) without using the SRT or other contractions hypotheses. The conventional methods, which use the SRT, for deriving the electric and magnetic field, generated by a point charge moving with constant velocity \mathbf{v} , are based on using coulomb's law in the rest frame of the point charge. So if we consider a charge Q located at \mathbf{r}_1 and a second q located at \mathbf{r}_2 . Let \hat{R} a unit vector in the direction of $(\mathbf{r}_1 - \mathbf{r}_2)$ and $r = |\mathbf{r}_2 - \mathbf{r}_1|$. Coulomb's law states that the force on q due to Q is given by:

$$\mathbf{F} = \frac{qQ\hat{R}}{4\pi\epsilon_0 r^2}$$

The electric field equals:

$$\mathbf{E} = \frac{\mathbf{F}}{q} = \frac{Q\hat{R}}{4\pi\epsilon_0 r^2}$$

Let now the two charges q and Q be at rest in a reference frame S' which has speed u_{ox} and moves uniformly to another reference frame S . The charge Q in the frame S' produces an electric field applied on q its components:

$$E'_x = \frac{Qx'}{4\pi\epsilon_0 r'^3}, \quad E'_y = \frac{Qy'}{4\pi\epsilon_0 r'^3}$$

since $x' = r' \cos \theta'$, $y' = r' \sin \theta'$ in spherical polar coordinates. So previous equations become:

$$E'_x = \frac{Q}{4\pi\epsilon_0 r'^2} \cos \theta', \quad E'_y = \frac{Q}{4\pi\epsilon_0 r'^2} \sin \theta' \quad (18)$$

Thus the electric field \mathbf{E} in the frame S' is symmetric as usual.

$$\mathbf{E}' = \frac{Q}{4\pi\epsilon_0 r'^3} \mathbf{r}' \quad (19)$$

The electric field \mathbf{E} in reference frame S determined as follow:

The same event is characterized by the momentum- energy of the photon in frame S and S' but not by the space-time coordinates [10, 11]. As we know that the interactions between two charges q and Q based on the exchange of photons. The charge Q in the frame S' emits a photon moves in a direction that making an angle θ' with positive ox' axis. This photon is viewed by the observer in frame S at an angle θ relative to the ox axis. The momentum of the photon in frame S' has components

$$P'_x = P' \cos \theta' , P'_y = P' \sin \theta' \quad (20a)$$

According to the relativity principle, Eq. (20a) could be written as

$$P_x = P \cos \theta , P_y = P \sin \theta \quad (20b)$$

Since the charge Q is at rest in a reference frame S' , and then Eq. (5) has the form:

$$P'_x = \gamma P_x , P'_y = P_y \quad (21)$$

Substituting Eq. (21) in Esq. (20) we then have:

$$\cos \theta' = \frac{p'_x}{P'} = \frac{\gamma P_x}{P'} = \frac{\gamma P \cos \theta}{P'} \quad (22a)$$

$$\sin \theta' = \frac{p'_y}{P'} = \frac{P_y}{P'} = \frac{P \sin \theta}{P'} \quad (22b)$$

As well as

$$\begin{aligned} P' &= \sqrt{p'^2_x + p'^2_y} \\ &= p \sqrt{\gamma^2 \cos^2 \theta + \sin^2 \theta} \\ &= \gamma p \sqrt{1 - \frac{u^2}{c^2} \sin^2 \theta} \end{aligned} \quad (23)$$

And by inserting Eq. (23) in formula (22) we will find

$$\cos \theta' = \frac{\cos \theta}{\sqrt{1 - \frac{u^2}{c^2} \sin^2 \theta}} \quad (24a)$$

$$\sin \theta' = \frac{\sin \theta}{\gamma \sqrt{1 - \frac{u^2}{c^2} \sin^2 \theta}} \quad (24b)$$

To get the same results in the textbook and reference we must

exchange P, P' by r, r' in Eq.(23), we get

$$r' = \gamma r \left(1 - \frac{u}{c} \cos \theta\right) \quad (25)$$

To determine the electric and magnetic field in reference frame S we take the inverse equations for Eqs.(6), and for our case we have:

$$E_x = E_x', \quad E_y = \gamma E_y', \quad B_x = 0, \quad B_z = \gamma \frac{u}{c^2} E_y' \quad (26)$$

Now substituting values of E_x' and E_y' from relation (18) in the last

equation we find

$$E_x = E_x' = \frac{Q \cos \theta'}{4\pi\epsilon_0 r'^2}, \quad E_y = \gamma E_y' = \frac{Q \gamma \sin \theta'}{4\pi\epsilon_0 r'^2}$$

Thus, by substituting Eqs. (24) and (25) in the last equation we obtain

$$E_x = E_x' = \frac{Q \cos \theta \left(1 - \frac{u^2}{c^2}\right)}{4\pi\epsilon_0 r^2 \left(1 - \frac{u^2}{c^2} \sin^2 \theta\right)^{\frac{3}{2}}}, \quad E_y = \frac{Q \sin \theta \left(1 - \frac{u^2}{c^2}\right)}{4\pi\epsilon_0 r^2 \left(1 - \frac{u^2}{c^2} \sin^2 \theta\right)^{\frac{3}{2}}}$$

Therefore, the electric field \mathbf{E} does not symmetric as in Eq.(19), i.e.:

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^3} \frac{1 - u^2/c^2}{\left(1 - \frac{u^2}{c^2} \sin^2 \theta\right)^{\frac{3}{2}}} \mathbf{r} \quad (27a)$$

The magnetic induction detected in frame S according to Eqs.(26) are:

$$B_z = \gamma \frac{u}{c^2} E'_y = \gamma \frac{u}{c^2} \frac{Q \sin \theta'}{4\pi\epsilon_0 r'^2}$$

$$B_x = 0, \quad = \frac{u/c^2 Q \sin \theta (1 - \frac{u^2}{c^2})}{4\pi\epsilon_0 r^2 (1 - \frac{u^2}{c^2} \sin^2 \theta)^{\frac{3}{2}}}$$

Or equivalently

$$\mathbf{B} = \frac{1}{c^2} \frac{Q(1 - \frac{u^2}{c^2})}{4\pi\epsilon_0 (1 - \frac{u^2}{c^2} \sin^2 \theta)^{\frac{3}{2}}} \mathbf{v} \times \mathbf{r} \quad (27b)$$

Eq.(27b) is the relativistic Biot- Savart law derived in this paper without SRT or LT. The usual claim is that magnetism is a relativistic effect. But in this paper is not true since the same result is obtained without assuming the LT or any other relativistic assumptions. Eq.(27b) is also equivalent to Eq.(17b), i.e.:

$$\mathbf{B} = \frac{\mathbf{v} \times \mathbf{E}}{c^2} \quad (28)$$

Example 3:

We consider a capacitor C, charged with electrical charges Q and -Q respectively. Now the field transformations in our method i.e. Esq.(5), allow us to work out the fields generated by a capacitor C moving with a constant velocity. Choosing the x-axis to be in the direction of motion of the capacitor C, we can set capacitor C at rest at the origin of the S frame. In this frame there is no magnetic field, and the electric field is simply

$$E_x = E_x^0 = \frac{Q}{\epsilon_0 \epsilon_r A_{yz}}$$

$$E_y = E_y^0 = \frac{Q}{\epsilon_0 \epsilon_r A_{xz}} \quad (29)$$

$$E_z = E_z^0 = \frac{Q}{\epsilon_0 \epsilon_r A_{xy}}$$

Where A represent the surface of one of the plates.

The relation (29) i.e. $E_y = E_y^0 = \frac{Q}{\epsilon_0 \epsilon_r A_{xz}}$ on which B. Rothenstein [12] assumed that the electric field in frame S' contracts according to Lorentz-Fitzgerald hypothesis, i.e.; $E'_y = \gamma E_y = \frac{\gamma Q}{\epsilon_0 \epsilon_r A_{xz}}$ led him to the relativistic electromagnetic field transformations,

$$E'_y = \gamma(E_y - uB_z) \quad , \quad B'_z = \gamma\left(B_z - \frac{u}{c^2}E_y\right).$$

We now assume that the capacitor C is parallel to the XZ-plane and moves with constant velocity v_x parallel to the X-axis. In frame S , it creates an electric field \mathbf{E} and a magnetic field \mathbf{B} oriented along the OY and the OZ axes, respectively. We can obtain the electric field E'_y and a magnetic field B'_z measured from frame S' by using the transformation equation (6), i.e.:

$$E'_y = \gamma(E_y - uB_z) \quad \text{(a)} \quad , \quad B'_z = \gamma\left(B_z - \frac{u}{c^2}E_y\right) \quad \text{(b)} \quad (30)$$

and according to Eq.(28), that the magnetic field of any charge distribution moving with uniform velocity v in the reference frame S is connected with the electric field of this distribution by the relation:

For this case, we have

$$B_z = \frac{v_x E_y}{c^2} \quad (31)$$

Substituting Eq.(31) in Eq.(30) , we get

$$E'_y = \gamma E_y \left(1 - \frac{uv_x}{c^2}\right)$$

Writing Eq.(13) for the cases $v = (v_x, 0, 0)$, and $v' = (v'_x, 0, 0)$, so Eq.(13) becomes:

$$\frac{1}{\sqrt{1 - \frac{v'^2_x}{c^2}}} = \frac{\left(1 - \frac{uv_x}{c^2}\right)}{\sqrt{1 - \frac{u^2}{c^2}} \sqrt{1 - \frac{v_x^2}{c^2}}} \quad (32)$$

Using Eq.(32) in the last relation, we have

$$E'_y = E_y \frac{\sqrt{1 - \frac{v_x^2}{c^2}}}{\sqrt{1 - \frac{v_x'^2}{c^2}}} \quad (33)$$

As is well known, one can consider moving capacitor C to be associated with its own frame of reference, so one can now consider that the frame S' to be co-moving with the capacitor C. Therefore $v'_x = 0$, when $E'_y = E_y^0$. Hence Eq.(33) can be written as:

$$E_y = \frac{E_y^0}{\sqrt{1 - \frac{v_x^2}{c^2}}} \quad (34a)$$

According to relativity principle, one can also consider frame S to be co-moving with the capacitor C,

$$E'_y = \frac{E_y^0}{\sqrt{1 - \frac{v_x'^2}{c^2}}} \quad (34b)$$

Where, $E_y^0 = \frac{Q}{\epsilon_0 \epsilon_r A_{xz}}$ is an electric field created from capacitor C at rest.

The relation (34b) is the same relation given by B. Rothenstein [12] which was assumed through the Lorentz-Fitzgerald hypothesis.

Now from Eq.(31) and Eq.(30b) we have:

$$B'_z = \frac{\gamma}{c^2} E_y (v_x - u)$$

Using Eqs.(4a and 34a) in the last relation, we obtain

$$B'_z = \frac{\gamma E_y^0 (1 - \frac{uv_x}{c^2}) v'_x}{c^2 \sqrt{1 - \frac{v_x^2}{c^2}}}$$

Finally, using Eq.(32) we get

$$(34c) B'_z = \frac{E_y^0 v'_x}{c^2 \sqrt{1 - \frac{v_x^2}{c^2}}} = \frac{E'_y v'_x}{c^2}$$

We see from Eqs.(34) that this approach excludes the role of the relativistic length contraction, and, the Lorentz-Fitzgerald contraction hypothesis is only a consequence of using the Lorentz's real force.

Conclusion

After rejecting the reality of ether, the relativistic length contraction in SRT was no longer the result of certain forces (real or fictitious), and didn't need any additional propositions concerning the structure of matter or the nature of the ether. In short, relativistic length contraction became a kinematics effect. The textbook methods for studying the well-known examples in electrodynamics are based on two hypotheses: the Lorentz-Fitzgerald contraction, and the relativistic length contraction. Even though the Lorentz-Fitzgerald contraction was an ad-hoc and theoretically unproved, so might this method be considered which provides some theoretical justification? Thus, regardless of whatever else one may believe about relativistic length contraction, we are led to believe that the Lorentz-Fitzgerald contraction is real.

References

- 1- A. Einstein, Ann. Phys. **17**, 891 (1905).
- 2- J. Terrell, Phys. Rev. **116**. 1041-1045 (1959).
R. A. Sorensen, Am. J. Phys. **63**, 413-415 (1995).
H. E. Wilhelm, Hadronic J. **19**, 1-39 (1996),
H. E. Wilhelm, phys. Ess. **6**, 382 – 398 (1993).
J. Hafele and R. Keating, Science **177**, 166 (1972).
C. Hackman and D. B. Sullivan, Am. J. phys. **63**, 306 – 317 (1995).
- 3- M. Harada and M. Sachs, phys. Ess. **11**, 521 – 523 (1998).
M. Harada, phys. Ess. **12**, 368 – 370 (1999).
O. D. Jefimenko, Z. Naturforsch. **53a**, 977-982 (1998).
C. K. Whitney, “How Can Paradox Happen?”, prepared for Seventh Conference on Physical Interpretations of Relativity Theory - London, 15-18 September (2000).
- 4- N. Hamdan, “Abandoning the Ideas of Length Contraction and Time Dilation”, “Galilean Electrodynamics” , **14**,83-88 (2003).
- 5- N.Hamdan,“Abandoning The Idea of Relativistic Length Contraction in Relativistic Electrodynamics”,“Galilean Electrodynamics” **15** , 71-75 (2004).
- 6- T Ton, "On the time dependent, generalization Coulomb and Biot-Savart laws," Am. J. Phys.**59**, pp.520-528 (1991).
- 7- R. Resnick, Introduction to special relativity, John Willey and Sons Inc. New York, 1968) pp. 167-177.
- 8- W. Rosser, Classical Electromagnetism via Relativity, (Butterworth London 1968) pp.34-38.
- 9 - S Hajra & A. Ghosh,“ Collapsc of SRT1”, Galilean Electrodynamics Vol. **16** , No. 4 pp.63-70 (2005).
- 10- [7c]- N. Hamdan, “On the Interpretation of the Doppler Effect in the Special Relativity (SRT)", Galilean Electrodynamics **17**, 29-34 (2006).
- 11- N. Hamdan, 2005. Derivation of the relativistic Doppler effect from the Lorentz force. Apeiron **12**: 47-61. (2005).
- 12 - B. Rothenstein, I. Zaharie,Los Alamos-archive Phys./0306059 (2003).