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# Lorentz Transformations Are Unable to Describe the Relativistic Doppler Effect

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## Abstract

We know paradoxes do not exist in nature and a complete theory does not include them. We only need to use our intuition to see that there are no logical inconsistencies. In this context, the relationship between the relativistic Doppler Effect(RDE) and Lorentz Transformations(LT) exhibits logical inconsistencies. It means that LT misrepresents reality and describes no physical effects.

In this paper we will explain how to eliminate such logical inconsistencies (contradictions).

## 1-Introduction

In his well–known article on "Special Relativity" Einstein succeeded in deriving LT [1], and then deriving the relativistic Doppler relations based on these transformations. Therefore RDE is related to the time dilation effect[2]. SRT accounts for various kinematical effects, like length contraction and time dilation. Several questions arise when examining this kinematical effect and many contradictions exist. Moreover SRT and relativistic Doppler relations are incompatible. In this regard we will see that the asymmetry of kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely.

In this paper, we depend on the method in [3,4], in which it is shown that the Doppler calculation procedure as well as interpretation is possibly only with the help of the Lorentz force law and the relativity principle. This will end the role of the Lorentz transformation (LT) and of time dilation in RDE.

## 2-Einstein's Method in Deriving Doppler's Formula :

It is well known that the color of light rays coming out of a moving source towards the observer tend to be blue shifted (i. e. high frequency), whereas a ray exiting a moving source in the opposite direction tends to be red shifted (i.e. low frequency).

The diversity of possibilities along with the existence of ether between the source and the observer leads to four possibilities, explained as follows :

1. If the source is receding/approaching from the rest observer, the frequency that observer sees is classically

$$f' = \frac{f_0}{1+(u/c)} \quad , \quad f' = \frac{f_0}{1-(u/c)} \quad (1a, 1b)$$

2. If the observer receding/approaching from the rest source, the frequency that observer sees is classically

$$f' = f_0\,(1-\frac{u}{c}) \quad , \quad f' = f_0\,(1+\frac{u}{c}) \quad (2a, 2b)$$

By excluding the idea of ether, Einstein has reduced these four possibilities to only two, namely :

$$f' = \sqrt{1-\frac{u^2}{c^2}}\,\frac{f_0}{1+\frac{u}{c}} = f_0\sqrt{\frac{1-u/c}{1+u/c}} \quad (3a) \quad , \quad f' = \sqrt{1-\frac{u^2}{c^2}}\,\frac{f_0}{1-\frac{u}{c}} = f_0\sqrt{\frac{1+u/c}{1-u/c}} \quad (3b)$$

Relativity principle makes it easier to use when it considers these two possibilities as actually one single possibility, and we get the second possibility through converting the speed sign in the first.

We can see the difference between the classical Doppler effect applied to light waves and the RDE. It makes no sense to talk about the velocity of either the source or the observer relative to the medium as one does in ether. One considers only the relative velocity $u$ between the source and the observer. As the RDE includes time dilation, i.e. the RDE includes also the transverse Doppler effect(TDE).

Einstein obtained in his work [1] the following formula:

$$f' = f_0\,\frac{\sqrt{1-(u^2/c^2)}}{1+(u/c)\cos\theta} \quad (4)$$

Where $u$ is the speed of source, $\theta$ is the direction of travel.

There are two particular cases that lead to simplifications. The first is motion along the line of sight-the longitudinal relativistic Doppler effect, where.

1- $$f' = \sqrt{1-\frac{u^2}{c^2}}\,\frac{f_0}{1+\frac{u}{c}} = f_0\sqrt{\frac{1-u/c}{1+u/c}} \quad (5a)$$

Had the source been approaching from the observer then

2- $$f' = \sqrt{1-\frac{u^2}{c^2}}\,\frac{f_0}{1-\frac{u}{c}} = f_0\sqrt{\frac{1+u/c}{1-u/c}} \quad (5b)$$

The other special case is that of transverse motion across the line of sight. In this case

3- $$f' = f_0\sqrt{1-\frac{u^2}{c^2}} = \frac{f_0}{\gamma} \quad (5c)$$

Eqs(5a,5b) have classical analogues in Eqs.(1a,1b).

The frequency is red shifted due to the dilation of the source time, Eq.(5c), and this effect(TDE) corresponds to the time slowing down on the source moving clock.

In SRT's formalism the key effect for RDE is time dilation, which plays an important part in modern physics. Therefore Eq.(5c) is considered a unique feature of SRT and is related to the dilation of time only for the moving source. Einstein believed that the general formula Eq.(4) which he deduced is an appropriate formula for the two cases, the source is moving and observer is at rest, or the source is at rest and observer is moving.

As remarked in Einstein's method, the RDE and TRD modes treat only source receding/approaching from the rest observer. However, an accurate analysis of Eq.(4) by using LT would reveal that there is an important contradiction between RDE and LT.

According to relativity principle, Eq.(4) could be written for the case of observer in motion as,

$$f' = f_0\,\gamma(1-\frac{u}{c}\cos\theta)$$

If the motion is normal to the line connecting source and observer, we then obtain from the last equation

$$f' = \gamma f_0$$

This equation shows a time contraction, instead of a time dilation as in Eq.(5c). We know that time contraction does not exist in SRT but the symmetry effect of TDR requires a time contraction.

We will see now that the asymmetry of kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely.

**3- Derivation of RDE and TDE from Lorentz Transformations**

Assume two inertial frames $S$ and $S'$, a source with frequency $f_0$ in the moving frame $S$, an observer in the rest frame $S'$, and the source approaching with the relative velocity $u \| ox$ from the observer.

The position of the radiation frequency of moving source is described by

$$x = ct \quad , \quad x' = ct' \tag{6}$$

where $(x,\ t)$ and $(x', t')$ are spatial and time intervals.
Then, we apply LT

$$x' = \gamma(x - ut) \quad , \; t' = \gamma(t - \frac{u}{c^2} x) \tag{7}$$

we have

$$t' = \gamma t (1 - \frac{u}{c}) \tag{8}$$

The frequency is derived as the inverse of time, i.e.:

$$t = \frac{1}{f_0} \quad , \quad t' = \frac{1}{f'} \tag{9}$$

If we insert Eq.(9) in (8), we 'll find

$$f' = \frac{f_0 \sqrt{1 - (u^2/c^2)}}{1 - (u/c)} = f_0 \sqrt{\frac{1 + (u/c)}{1 - (u/c)}} \tag{10}$$

Had the source been receding from the observer, then by replacing $u$ with $-u$ in Eq.(17), we have

$$f' = f_0 \sqrt{\frac{1 - (u/c)}{1 + (u/c)}} \tag{11}$$

Let the reference time in $S$ be $t_0$. The reference time in $S'$ is defined by Eq.(7), i.e.:

$$t' = \gamma t_0 \qquad (12)$$

Using (9) in (12), we get

$$f' = \frac{f_0}{\gamma} = f_0 \sqrt{1-(u^2/c^2)} \qquad (13)$$

Eq.(10,11 and 13) are the relativistic Doppler shift derived by Einstein [1], but now is derived from LT directly.

According to relativity principle , we can also consider the frame $S$ to be co-moving with the source and receding /approaching the observer. Then Eq. (7) could be written as

$$x = \gamma(x' + u t') \quad , \; t = \gamma(t' + \frac{u}{c^2} x') \qquad (14)$$

Inserting (6) in (14), we obtain

$$t = \gamma t' (1 + \frac{u}{c})$$

then using (9) in the last equation, we the have

$$f_0 = \frac{f' \sqrt{1-(u^2/c^2)}}{(1 + \frac{u}{c})}$$

Or

$$f' = \frac{f_0 \,(1 + \frac{u}{c})}{\sqrt{1-(u^2/c^2)}}$$

$$= f_0 \sqrt{\frac{1 + u/c}{1 - u/c}} \qquad (15)$$

If the observer is receding from the source, then by replacing $u$ with $-u$ in Eq.(15), we have

$$f' = f_0 \sqrt{\frac{1-(u/c)}{1+(u/c)}} \qquad (16)$$

If the motion is normal to the line connecting source and observer, we then obtain

$$f' = \frac{f_0}{\sqrt{1-\frac{u_T^2}{c^2}}} = \gamma f_0 \qquad (17)$$

where in normal movement, the radial component is zero, $u_r = 0$, and since

$$u^2 = u_t^2 + u_r^2 = u_t^2$$

We obtain also a change in frequency as per Eqs. (17). So the Doppler effect exists even though there is no component of relative motion along the line of sight.

The reference time of the moving observer becomes long i. e. according to Eqs.(9) and (17), we have

$$t' = t_0 / \gamma \qquad (18)$$

The reference time of the moving observer decrease(time contraction), thus the frequency of the light source that is seen by the moving observer increases as in Eq.(15) and decreases as in Eq.(16).

Eqs.(15,16 and 17) do not derived by Einstein because it need to time contraction as in Eq.(18) and the kinematical effect derived by LT is time dilation but not time contraction.

Thus, the Lorentz transformation (7) and (14) have been used for the calculation of the so called relativistic effect, and we now know that they give the possibility to calculate the relativistic effect only if LT has time contraction as in Eq.(18).

It means that LT misrepresents reality and reflects no physical effects [3,4].

**4- Derivation of RDE and TDE from Lorentz Force**

Now assume that a particle q in frame $S$ emits a light wave (photon) that moves in a direction that makes an angle $\theta$ with the positive ox axis. The light is received at the observer in frame $S'$ at an angle $\theta'$ relative to the $ox'$ axis. In [3,4] we have derived the following relation:

$$v' = \gamma v(1 - \frac{u}{c}\cos\theta) \qquad (19a)$$

But the connection between the two frequencies in frames $S$ and $S'$ is given also by

$$v = \gamma v'(1 + \frac{u}{c}\cos\theta') \qquad (19b)$$

If we consider that the frame $S$ to be co-moving with the source and receding /approaching observer, Eq. (19a) becomes

$$1 \text{ - } \theta = \theta' = 0^\circ \text{ i.e. } \nu' = \nu_0 \gamma (1 - \frac{u}{c}) = \nu_0 \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}} \tag{20a}$$

$$2 \text{ - } \theta = \theta' = 180^\circ \text{ i.e. } \nu' = \nu_0 \gamma (1 + \frac{u}{c}) = \nu_0 \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}} \tag{20b}$$

Eqs. (20a,20b) do not have relativistic analogues, but have classical analogues in Eqs. (2a,2c).

According to relativity principle , we can also consider the frame $S'$ to be co-moving with the observer and receding /approaching source, then Eq. (19b) could be written as

$$\nu' = \frac{\nu_0 \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c} \cos \theta'} \tag{19c}$$

Hence

$$1 \text{ - } \theta = \theta' = 0^\circ \text{ i.e. } \nu' = \frac{\nu_0 \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c}} = \nu_0 \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}} \tag{21c}$$

$$2 \text{ - } \theta = \theta' = 180^\circ \text{ i.e. } \nu' = \frac{\nu_0 \sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{u}{c}} = \nu_0 \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}} \tag{21d}$$

Eqs. (21c,21d) are identical to Eqs. (5a,5b) in SRT, and the classical analogues are (1a, 1b).

If the velocity of the observer/ source is perpendicular to the line of sight, then we have from Eq. (19a),

$$\theta = \theta' = 90^\circ \text{ i.e. } \nu' = \frac{\nu_0}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma \nu_0 \tag{22a}$$

And from Eq. (19c), we have

$$\theta = \theta' = 90^\circ \text{ i.e. } \nu' = \nu_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{\nu_0}{\gamma} \tag{22b}$$

In [3] we have shown, that had it not been for the existence of formula (22a) and (22b) together, there would be no equality between the two formulas (20a) and (21c), and the two formulas (20b) and (21d). This means that formula (22a) exists since this formula is not the outcome of SRT due to time contraction. This would mean that LT is unable to describe a well – known physical reality, namely Doppler effect.

We turn to Eqs.(19a,19c) i.e.

$$\nu' = \gamma\nu_0 (1 - \frac{u}{c}\cos\theta) \quad , \quad \text{and} \quad \nu' = \frac{\nu_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}\cos\theta'} \tag{23}$$

The two angles in Eqs.(23) differ when we include the effect of aberration. If we let $\theta$ denote the angle with respect to the source's frame and $\theta'$ denote the angle with respect to the observer's frame, then we have

$$\frac{\nu_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}\cos\theta'} = \gamma\nu_0 (1 - \frac{u}{c}\cos\theta)$$

Or

$$1+\frac{u}{c}\cos\theta' = \frac{1-\frac{u^2}{c^2}}{1-\frac{u}{c}\cos\theta} \quad \text{i. e.} \quad \cos\theta' = \frac{\cos\theta - \frac{u}{c}}{1-\frac{u}{c}\cos\theta} \tag{24}$$

Eq.(24) describe the aberration of light. Star aberration arises because the observe moves with the orbital speed of the Earth. If , as SRT asserts, the movement of the light source is equivalent to the movement of the observer, star aberration has to arise as in the case when the source moves. However, the observations of binary stars prove that there is no aberration when the stars move.

**Conclusion:**

Due to the Eqs.(22a) and (22b) together, the RDE formula for a moving observer can also be written in the form used for a moving source.

Certainly formula (22a), which does not have an equivalent in the SRT, is very significant for the formula (22b) for the equality of the longitudinal relativistic Doppler effect. That means the longitudinal relativistic Doppler effect for a moving observer can be also written in the form used for a moving source.

The kinematical effect derived by LT is time dilation but not time contraction. Thus, the asymmetry of the kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely. These contradictions cannot be removed without excluding LT and deriving RDE and TRD formula without LT [3,4].

Star aberration and the TDR arise as the result of using the Lorentz force and relativity principle. Both effects do not prove that time dilation takes place in moving systems. Both these phenomena contradict SRT and prove it is false.

**References**

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