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New Methods in Relativistic Electrodynamics without the Hypotheses of Contraction

N. Hamdan
Department of Physics
University of Aleppo
Aleppo- Syria
E-mail: nhamdan59@hotmail.com
nhamdan2@lycos.com

Abstract

We have shown that choosing a different set of postulates enables us to cancel the Lorentz Transformation (LT) from the main body of special relativity theory (SRT). Hence, this approach excludes the role of relativistic length contraction and as a result, the Lorentz-Fitzgerald contraction hypothesis is only a consequence of using Lorentz's real force. We use well-known examples to confirm that contraction hypotheses are not required in relativistic electrodynamics.

Key words: Lorentz force, Lorentz-Fitzgerald contraction, Relativistic length contraction

1 – Introduction

Lorentz deduced LT to solve the following problem: In passing from an absolute rest frame k (The Ether frame, according to the prevailing wisdom) to another inertial frame k' moving with constant velocity u with respect to k, Maxwell's equations change form under Galilean transformations, acquiring additional first and second degree terms in $\frac{u}{c}$. Lorentz noticed that the first-order term in $\frac{u}{c}$ could easily be eliminated by the following alteration ($t' = t \quad \Rightarrow \quad t' = t - \frac{u}{c}x$) where $\gamma = 1\Big/\left(1 - u^2/c^2\right)^{\frac{1}{2}}$. But, Lorentz could not explain the null result of the Michelson – Morely experiment without using the contraction hypothesis and at the same time get rid of the second-order term in $\frac{u}{c}$. He accepted the hypothesis of Fitzgerald that moving bodies contract i.e., the equations($x' = \gamma(x - ut)$, $y' = y$, $z' = z$), are real for moving electromagnetic bodies. Lorentz did not prove the real contraction of his transformation equation. The explanation of the Lorentz - Fitzgerald contraction was that because of an interaction with the ether, all bodies contracted in the direction of their motion relative to the ether by the factor, $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$.

This hypothesis is important, especially with regard to many calculations in classical electrodynamics. In his paper Einstein [1], derived LT to justify his two postulates. He claimed that the postulates are absolutely real, base on the assumption that LT and its kinematical effects are general laws of Nature.

Einstein showed that the Lorentz - Fitzgerald contraction, which had been introduced previously into classical electrodynamics, is a simple consequence of LT. So after denying the ether, Einstein accepted the contraction hypothesis as the kinematical length contraction of a moving object. According to the Lorentz-Fitzgerald contraction the charge density contracted in the direction of the motion by

$$\rho = \rho_0 (1 - \frac{v^2}{c^2})^{-1/2} \tag{1}$$

In Eq.(1), the quantity ρ_0 is the charge at rest. The methods for deriving Eq.(1) in relativistic electrodynamics textbooks are based on LT and the relativistic length contraction. However, the length of a moving body has never been compared with the length of a body at rest and the hypothesis of "length contraction" in relativistic electrodynamics has been the subject of considerable controversy and re-interpretation [2]. Moreover, to this day it remains a controversial aspect of Einstein's theory [3,4b].

In my papers [4], I suggest another way to account for the kinematics effects in relativistic electrodynamics. This method does not use LT for a charged particle and Maxwell fields; instead, it includes Lf within the main body of SRT,i.e.;

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \ , \tag{2}$$

and applies the relativity principle (that the laws of Physics have the same formulation relative to any inertial system) rather than the special relativity principle (that the laws of physics are invariant under LT).

Using only the Lf and the relativity principle, instead of Maxwell's equations and LT, we derive the fundamental relativistic equations. We used a well-known example to show that the hypothesis of contraction is not required to derive Eq.(1). In this paper, we depend on the method shown in [4b] in order to generalize the example mentioned at the end of the paper. Then we clarify that the example in [3] can be studied according to our method. Thus in the two examples contained in this paper, we show that they may be studied without the use of the famous hypotheses of contraction.

2 - Relativistic Transformation Relations:

As demonstrated also in my papers of [4a], contrary to what is often claimed in SRT presentations, the relativistic transformation relations are produced without LT and its kinematical effect. We derive the relativistic transformation relations for force, velocity and the relativistic electromagnetic field transformations as well as the relativistic factor γ :

$$F'_x = F_x - \frac{\frac{u}{c^2} F_y v_y}{\left(1 - \frac{u v_x}{c^2}\right)} - \frac{\frac{u}{c^2} F_z v_z}{\left(1 - \frac{u v_x}{c^2}\right)} \ \text{(3a)}, \quad F'_y = \frac{F_y}{\gamma\left(1 - \frac{u v_x}{c^2}\right)} \ \text{(3b)}, \quad F'_z = \frac{F_z}{\gamma\left(1 - \frac{u v_x}{c^2}\right)} \ \text{(3c)}$$

$$v'_x = \frac{v_x - u}{1 - \frac{u v_x}{c^2}} \ \text{(4a)} \ , \quad v'_y = \frac{v_y}{\gamma\left(1 - \frac{u v_x}{c^2}\right)} \ \text{(4b)}, \quad v'_z = \frac{v_z}{\gamma\left(1 - \frac{u v_x}{c^2}\right)} \ \text{(4c)}$$

Where $\gamma = 1 \Big/ \left(1 - u^2/c^2\right)^{\frac{1}{2}}$.

As well as

$$E'_x = E_x \quad , \quad E'_y = \gamma(E_y - uB_z) \ , \quad E'_z = \gamma(E_z + uB_y)$$

$$B'_x = B_x \quad , B'_y = \gamma\left(B_y + \frac{u}{c^2} E_z\right) \ , \quad B'_z = \gamma\left(B_z - \frac{u}{c^2} E_y\right) \tag{5}$$

By this approach we also get the Lorentz contracted charge density, i.e.;

$$\rho = \rho_0 (1 - \frac{v^2}{c^2})^{-1/2} \qquad (6)$$

This was done without using the hypothesis proposed by Lorentz–Fitzgerald or relativistic length contraction [4b].

3 - Examples
3 –a:
Let us take a cylindrical charged beam of radius (a) in which the charge density is ρ (line charge density $\lambda = \pi a^2 \rho$), wherein all charged particles travel in frame s with velocity ***v*** along its length. We will try to calculate the force that is produced by the charges on a particle of charge q on the surface of the beam, which moves with the same velocity as the charges. The electric field and the magnetic field produced by the moving beam of charges at the location of q are:

$$E = \frac{\lambda}{2\pi a \varepsilon_0} = \frac{\rho a}{2\varepsilon_0} \quad \text{(a)}, \qquad B = \frac{v\ \rho a}{2\varepsilon_0 c^2} \quad \text{(b)} \qquad (7)$$

Hence, the Lorentz force on q is

$$F = \frac{qa\rho}{2\varepsilon_0}(1 - \frac{v^2}{c^2}) \qquad (8)$$

Now, according to the relativity principle, the transformed force in frame s' is:

$$F' = \frac{qa\rho'}{2\varepsilon_0}(1 - \frac{v'^2}{c^2}) \qquad (9)$$

Where $c^2 = 1/\varepsilon_0 \mu_0$

On the other hand, if we take the relativistic transformation equation for the perpendicular component of the force, Eq.(3), i.e.;

$$F' = \frac{F}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad (10)$$

and substituting (8) into (10), we obtain for the force acting on q in the frame s',

$$F' = \frac{q\rho a(1 - \frac{v^2}{c^2})}{2\varepsilon_0 \gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad (11)$$

Now, using the following relation [4b]:

$$\frac{1}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{\left(1-\frac{uv_x}{c^2}\right)}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}} \tag{12}$$

in Eq.(11), we get

$$F' = \frac{q\rho a}{2\varepsilon_0}(1-\frac{v'^2}{c^2})^{1/2}(1-\frac{v^2}{c^2})^{1/2}$$

The last relation is equivalent to the following relation

$$F' = \frac{q\rho a(1-\frac{v^2}{c^2})^{1/2}}{2\varepsilon_0(1-\frac{v'^2}{c^2})^{1/2}}(1-\frac{v'^2}{c^2}) \tag{13}$$

Now by comparing Eq.(13) with the Eq.(9), we have

$$\rho' = \frac{\rho(1-\frac{v^2}{c^2})^{1/2}}{(1-\frac{v'^2}{c^2})^{1/2}} \tag{14}$$

The relation (14) leads to the following relations [4b]:

$$\rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}} \text{ (a)}, \qquad \rho' = \frac{\rho_0}{\sqrt{1-\frac{v'^2}{c^2}}} \text{ (b)} \tag{15}$$

In classical electrodynamics, one postulates the Lorentz - Fitzgerald contraction to derive Eqs.(15), while in relativistic electrodynamics, one uses the relativistic length contraction for the same equations. We have deduced them without using any of the afore-mentioned hypotheses.

3 –b

We consider a capacitor C, charged with electrical charges Q and –Q respectively. Now the field transformations in our method i.e. Eqs.(5), allow us to work out the fields generated by a capacitor C moving with a constant velocity. Choosing the x-axis to be in the direction of motion of capacitor C, we can set capacitor C at rest at the origin of the s frame. In this frame there is no magnetic field, and the electric field is simply

$$E_x = E_x{}^0 = \frac{Q}{\varepsilon_0\varepsilon_r A_{yz}}, \quad E_y = E_y{}^0 = \frac{Q}{\varepsilon_0\varepsilon_r A_{xz}} \ \textbf{(16)}, \ E_z = E_z{}^0 = \frac{Q}{\varepsilon_0\varepsilon_r A_{xy}}$$

Where A represent the surface of one of the plates.

The components of the fields in s' now follow from the field transformations of Eqs.(5). We have

$$E'_x = E_x = \frac{Q}{\varepsilon_0\varepsilon_r A_{yz}}, \quad E'_y = \gamma E_y = \frac{\gamma Q}{\varepsilon_0\varepsilon_r A_{xz}}, \quad E'_z = \gamma E_z = \frac{\gamma Q}{\varepsilon_0\varepsilon_r A_{xy}}$$

and

$$B'_x = 0,\ \ B'_y = \gamma \frac{u}{c^2} E_z = \frac{u}{c^2} E'_z\ ,\ \ \ B'_z = \gamma \frac{u}{c^2} E_y = \frac{u}{c^2} E'_y$$

Or equivalently

$$' = \frac{' \times \ '}{c^2} \tag{17}$$

The relation (16), i.e. $E_y = E_y{}^0 = \dfrac{Q}{\varepsilon_0 \varepsilon_r A_{xz}}$ in which B. Rothenstein [3] assumed that the electric field in frame s' contracts according to Lorentz-Fitzgerald hypothesis, $E'_y = \gamma E_y = \dfrac{\gamma Q}{\varepsilon_0 \varepsilon_r A_{xz}}$ led him to the relativistic electromagnetic field transformations,

$$E'_y = \gamma\left(E_y - uB_z\right)\ ,\ B'_z = \gamma\left(B_z - \frac{u}{c^2} E_y\right).$$

We now assume that capacitor C is parallel to the XZ-plane and moves with constant velocity v_x parallel to the X-axis. In frame s, it creates an electric field   and a magnetic field oriented along the OY and the OZ axes respectively. We can obtain the electric field E'_y and a magnetic field B'_z measured from frame s' by using the transformation equation of (5),

$$E'_y = \gamma\left(E_y - uB_z\right) \text{ (a) },\quad B'_z = \gamma\left(B_z - \frac{u}{c^2} E_y\right) \text{ (b)} \tag{18}$$

and by assuming, according to Eq.(17), that the magnetic field of any charge distribution moving with uniform velocity ***v*** in the reference frame s is connected with the electric field of this distribution by the relation:

$$= \frac{\times}{c^2}$$

For this case, we have

$$B_z = \frac{v_x . E_y}{c^2} \tag{19}$$

Substituting Eq.(19) in Eq.(18a) , we get

$$E'_y = \gamma E_y \left(1 - \frac{uv_x}{c^2}\right)$$

Writing Eq.(12) for the cases $v = (v_x, 0, 0)$, and $v' = (v'_x, 0, 0)$, Eq.(12) becomes:

$$\frac{1}{\sqrt{1 - \frac{v_x'^2}{c^2}}} = \frac{\left(1 - \frac{uv_x}{c^2}\right)}{\sqrt{1 - \frac{u^2}{c^2}}\sqrt{1 - \frac{v_x{}^2}{c^2}}} \tag{12a}$$

Using Eq.(12a) in the last relation, we have

$$E'_y = E_y \frac{\sqrt{1-\frac{v_x^{\,2}}{c^2}}}{\sqrt{1-\frac{v_x'^{\,2}}{c^2}}} \qquad (20)$$

As is well known, one can consider moving capacitor C to be associated with its own frame of reference, so one can now consider frame s' to be co-moving with capacitor C. Therefore, $v'_x = 0$, when $E'_y = E_y^{\ 0}$. Hence Eq.(20) can be written ,

$$E_y = \frac{E^0{}_y}{\sqrt{1-\frac{v_x^{\,2}}{c^2}}} \qquad (21a)$$

According to relativity principle, one can also consider frame s to be co-moving with capacitor C,

$$E'_y = \frac{E^0{}_y}{\sqrt{1-\frac{v_x'^{\,2}}{c^2}}} \qquad (21b)$$

Where, $E_y^{\ 0} = \dfrac{Q}{\varepsilon_0 \varepsilon_r A_{xz}}$ is an electric field created from capacitor C at rest.

The relations in (21) are the same relations given by B. Rothenstein [3] which were derived through the Lorentz-Fitzgerald hypothesis. Now from Eq.(18b) and Eq.(19) we have:

$$B'_z = \frac{\gamma}{c^2} E_y (v_x - u)$$

Using Eqs.(4a and21a) in the last relation, we obtain

$$B'_z = \frac{\gamma E^0{}_y (1-\frac{u v_x}{c^2}) v'_x}{c^2 \sqrt{1-\frac{v_x^{\,2}}{c^2}}}$$

Finally, using Eq.(12a) we get

$$\text{(21c)} \quad B'_z = \frac{E^0{}_y v'_x}{c^2 \sqrt{1-\frac{v_x'^{\,2}}{c^2}}} = \frac{E'_y v'_x}{c^2}$$

We see from Eqs.(15, and 21) that this approach excludes the role of the relativistic length contraction and that the Lorentz-Fitzgerald contraction hypothesis is only a consequence of using Lorentz's real force.

Conclusion

After rejecting the reality of ether, the relativistic length contraction in SRT was no longer the result of certain forces (real or fictitious), and didn't need any additional propositions concerning the structure of matter or the nature of the ether. In short, relativistic length contraction became a kinematic effect. The textbook methods for studying the well-known examples in electrodynamics are based on two hypotheses: the Lorentz-Fitzgerald contraction, and the relativistic length contraction. The basic idea of this paper is that they are not required. This was presented in papers [4] as a consequence of using the Lorentz real force.

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