A Novel Derivation of the Electric and Magnetic Fields of an Uniformly Moving Point Charge

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Abstract

A novel and a simple approach is presented for the derivation of electric field intensity and magnetic induction generated by a uniformly moving point charge. The derivation starts with Coulomb's law in a static frame of reference without using the usual transformation of space – time (i.e. Lorentz (LT) and fields. By this method we obtain the electric field intensity, $\mathbf{E}$ and magnetic induction, $\mathbf{B}$.

1- Introduction

Derivation of $\mathbf{E}$ and $\mathbf{B}$ generated by a point charge moving with constant velocity $\mathbf{v}$ was first obtained by O.Heaviside [1].

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^3}\frac{(1-\frac{u^2}{c^2})}{(1-\frac{u^2}{c^2}\sin^2\theta)^{3/2}}\mathbf{r}$$

$$\mathbf{B} = \frac{\mathbf{v}\times\mathbf{E}}{c^2} \qquad \text{(1b)}$$

Where $\mathbf{r}$ is the distance between the point where the measurement of $\mathbf{E}$ and $\mathbf{B}$ take place and the charge. θ represents the angle between $\mathbf{r}$ and $\mathbf{v}$.

Many methods appeared which derive the relations in (1), depending on the use of Maxwell's equations and the concept of retarded potential [2, 3, and 4]. New methods appeared after the emergence of special relativity theory (SRT). These methods, [5, 6, 7] are simpler than the old [2, 3 and 4]. All new methods depend on the use of LT and other relativistic transformation relations. Alternately, Jeffimenko could find approach [2], based on a general electromagnetic field equation without making use of the transformation equations.

In this paper we follow our method presented in [8, 9, and 10] to derive eqs (1) without using SRT or other contraction hypotheses. Thus, in contrast to conventional treatments, our approach is simple and novel.

2- Relativistic Transformation Relations Without SRT or LT.

As demonstrated in papers [8,9], contrary to what is often claimed in SRT presentations, the relativistic transformation relations are produced without LT and its kinematic effects.

Following the same approach used in [8,9], we can obtain the relativistic equations for force, momentum-energy and electromagnetic field transformations:

$$F'_x = F_x - \frac{\frac{u}{c^2} F_y v_y}{\left(1 - \frac{uv_x}{c^2}\right)} - \frac{\frac{u}{c^2} F_z v_z}{\left(1 - \frac{uv_x}{c^2}\right)} \text{ (2a)}, \quad F'_y = \frac{F_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \text{ (2b)}, \qquad F'_z = \frac{F_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \quad \text{(2c)}$$

$$P'_x = \gamma(P_x - \frac{u}{c^2}\varepsilon\ ) \text{ (3a)} \ , \quad \varepsilon' = \gamma(\varepsilon - uP_x) \ \text{ (3b)}, \quad P'_y = P_y \text{ (3c)}, \quad P'_z = P_z \text{ (3d)}$$

as well as

$$E'_x = E_x \quad , \quad E'_y = \gamma(E_y - uB_z) \ , \quad E'_z = \gamma(E_z + uB_y)$$
$$B'_x = B_x \quad , B'_y = \gamma\left(B_y + \frac{u}{c^2} E_z\right) \ , \quad B'_z = \gamma\left(B_z - \frac{u}{c^2} E_y\right) \qquad (4)$$

Where $\gamma = 1 \big/ (1 - u^2/c^2)^{\frac{1}{2}}$.

Now we can clarify in this paper that the re-derivation of relations (1) is possible without the use of the above methods.

3- The electric field intensity, $\mathbf{E}$ and Magnetic Induction, $\mathbf{B}$.

The conventional methods, which use SRT for deriving electric and magnetic fields generated by a point charge moving with constant velocity $\mathbf{v}$, are based on using coulomb's law in the rest frame of the point charge. So if we consider a charge Q located at $\mathbf{r_1}$ and a second q located at $\mathbf{r_2}$. Let $\hat{R}$ be a unit vector in the direction of ($\mathbf{r_1}$ - $\mathbf{r_2}$) and $r = |\mathbf{r_2} - \mathbf{r_1}|$. Coulomb's law states that the force on q due to Q is given by:

$$\mathbf{F} = \frac{qQ\hat{R}}{4\pi\varepsilon_0 r^2}$$

The electric field equals:

$$\mathbf{E} = \frac{\mathbf{F}}{q} = \frac{Q\hat{R}}{4\pi\varepsilon_0 r^2}$$

Now let the two charges q and Q be at rest in a reference frame S' which has speed u_{ox} and moves uniformly to another reference frame S . The charge Q in the frame S' produces a force $\mathbf{F}' = q\mathbf{E}'$ applied on q. Its components are:

$$F'_x = \frac{qQx'}{4\pi\varepsilon_0 r'^3} \ , \ F'_y = \frac{qQ\,y'}{4\pi\varepsilon_0 r'^3}$$

since $x' = r'\cos\theta'$, $y' = r'\sin\theta'$ in spherical polar coordinates .So the previous equations become:

$$F'_x = \frac{qQ}{4\pi\varepsilon_0 r'^2}\cos\theta' \ , \ F'_y = \frac{qQ}{4\pi\varepsilon_0 r'^2}\sin\theta' \quad (5)$$

The magnitude of the electric field, $\mathbf{E}$ in frame S' is symmetric as usual. As

$$F' = \sqrt{F'_x + F'_y} = \frac{qQ}{4\pi\varepsilon_0 r'^2}$$

So

$$E' = \frac{Q}{4\pi\varepsilon_0 r'^2} \qquad (6)$$

The force on q as measured in reference frame S is determined as follows:
The same event is characterized by the momentum-energy of the photon in frame S and S' but not by the space-time coordinates. We know that the interactions between two charges q and Q are based on the exchange of photons. The charge Q in frame S' emits a photon in a direction that makes an angle θ' with the positive ox' axis. This photon is viewed by the observer in frame S at angle θ relative to the ox axis. The momentum of the photon in frame S' has the components,

$$P'_x = P'\cos\theta' \ , P'_y = P'\sin\theta' \quad (7a)$$

According to relativity principle, Eq. (7a) could be written as

$$P_x = P\cos\theta \ , P_y = P\sin\theta \quad (7b)$$

but since $\varepsilon' = cP'$ and $\varepsilon = cP$, so Eqs.(7) become

$$P'_x = \frac{\varepsilon'}{c}\cos\theta' \ , \ P_x = \frac{\varepsilon}{c}\cos\theta \ \text{(8a)}, \ P'_y = \frac{\varepsilon'}{c}\sin\theta' \ , P_y = \frac{\varepsilon}{c}\sin\theta \ \text{(8b)}$$

Now if we insert Eq. (8a) in formula (3a and 3b) we will find

$$\varepsilon'\cos\theta' = \gamma\varepsilon(\cos\theta - \frac{u}{c}) \ , \ \varepsilon' = \gamma\varepsilon(1 - \frac{u}{c}\cos\theta)$$

By solving the two equations we find:

$$\cos\theta' = \frac{\cos\theta - \frac{u}{c}}{1 - \frac{u}{c}\cos\theta} \qquad (9a)$$

And by inserting Eq.(8b) in formula (3c) we will find

$$\frac{\varepsilon'}{c}\sin\theta' = \frac{\varepsilon}{c}\sin\theta \quad \text{(9b), i.e.} \quad \sin\theta' = \frac{\sin\theta}{\gamma(1-\frac{u}{c}\cos\theta)} \qquad \text{(9c)}$$

Once again using Eqs.(8b) in (3c) we have

$$P'\sin\theta' = P\sin\theta$$

Hence, from Eq.(9c) we have

$$P' = \gamma P(1-\frac{u}{c}\cos\theta)$$

To get the same results in the textbook and reference we must exchange P , P' by r , r' in the last relation,

$$r' = \gamma r(1-\frac{u}{c}\cos\theta) \qquad (10)$$

The force in frame S is

$$F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{F_x'^2 + F_y'^2/\gamma^2}$$

where we have used Eqs. (2a and 2b) in the last relation.

Now substituting values of F_x' and F_y' from relation (5) in the last equation we find,

$$F = \frac{qQ}{4\pi\varepsilon_0 r'^2}\sqrt{(\cos\theta')^2 + \frac{(\sin\theta')^2}{\gamma^2}}$$

and by substituting Eqs.(9a,9c) and (10) in the last equation we obtain,

$$F = \frac{qQ}{4\pi\varepsilon_0 r^2}\frac{1-u^2/c^2}{(1-\frac{u}{c}\cos\theta)^3}\sqrt{(1-\frac{u}{c}\cos\theta)^2}$$

$$= \frac{qQ}{4\pi\varepsilon_0 r^2}\frac{1-u^2/c^2}{(1-\frac{u}{c}\cos\theta)^2}$$

Therefore, the magnitude of the electric field $\mathbf{E}$ is not symmetric as in Eq.(6a),i.e.:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{1-u^2/c^2}{(1-\frac{u^2}{c^2}\cos\theta)^2} \qquad (11a)$$

The magnetic induction detected in frame S according to Eqs.(4) is:

$$B_x = 0, \qquad B_z = \gamma \frac{u}{c^2} E'_y = \gamma \frac{u}{c^2} \frac{Q \sin\theta'}{4\pi\varepsilon_0 r'^2} = \frac{u / c^2 Q \sin\theta}{4\pi\varepsilon_0 \gamma^2 (1 - \frac{u}{c}\cos\theta)^3}$$

Or equivalently

$$\mathbf{B} = \frac{1}{c^2} \frac{Q(1 - \frac{u^2}{c^2})}{4\pi\varepsilon_0 (1 - \frac{u}{c}\cos\theta)^3} \mathbf{u} \times \mathbf{r} \qquad (11b)$$

Eq.(11b) is the Biot- Savart law

Conclusion

The textbook methods for studying the well-known example in electrodynamics are based on the knowledge of transformation equations for coordinates, time and fields [5, 6, and 7]. Derivation [2,] do not use SRT. The basic idea of this paper is that the equations (1) can be derived without using SRT and LT for space and time coordinates. This example confirms our belief that the afore-mentioned famous methods found in electrodynamic textbooks are not required.

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