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Unchecked Aspects of Variation of Acceleration due to Gravity with Altitude

Ajay Sharma

Fundamental Physical Society. His Mercy Enclave. Post Box 107 GPO Shimla 171001 HP India

Email; ajay.pqr@gmail.com

Abstract

It is correctly established that g decreases with altitude, but the variation of g with atmospheric pressure (decreases with altitude) is not considered in precise experiments in the existing literature. Torricelli determined in pioneering experiments that height of mercury column in barometer as 0.76m due to atmospheric pressure in 1644. Newton formulated g in 1685, and then Pascal's Law was treated in presence of gravity for imaginary cylinder of liquid. Thus equation P=DgH is obtained which relates acceleration due to gravity, g with atmospheric pressure, P. The expression for variation in g with altitude as $g_h = g/(1+h/R)^2$, by both methods will be compared. At sea level the heights of liquid columns (for water 10.33m , for glycerine 8.202m , ethyl alcohol 13.16m) are independent of other factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity etc. At height of 2 km above the surface of the earth the heights of liquid columns are reduced e.g. for mercury 0.5967m, for water 8.1158m and for glycerine 6.4411 m. Now measuring P, H and g can be calculated. The value of g can be determined by both methods at various heights and should be same. Theoretically when atmospheric pressure becomes zero then value of g_H (P/DH) must tend to zero; according to $g_h = g/(1+h/R)^2$, g_h becomes zero at infinite large distances. But no such attempts have been reported in literature, hence it is open problem especially when tubes of various diameters are considered and characteristics of liquids are different. Due to diverse experimental conditions of liquids and equipments, mercury may be regarded as ideal liquid for such measurements of pressure. The value of g due to altitude decreases steadily, whereas due to atmospheric pressure g decreases abruptly. So sensitive experiments are absolutely necessary to draw concrete conclusions.

1.0 Introduction

Torricelli formed mercury barometer in 1644 and confirmed the height of mercury column as 0.76m and acceleration due to gravity was defined by Newton in 1685, then expression for pressure [1-3] was determined as

$P = DgH$ (1)

was calculated using Pascal law in presence of gravity. Here P is pressure; g is acceleration due to gravity and H is height of liquid column. Consider a liquid of density D is in equilibrium of rest. Eq.(1) is derived for imaginary cylinder of liquid. Consider a liquid of density D is in equilibrium of rest. Therefore net force acting cylinder will also be zero i.e.

$F_1 + Mg - F_2 = 0$ (2)

where F_1 is force acting vertically downwards on the top face of the cylinder, F_2 is the force is acting vertically upwards on the lower face of cylinder. Mg is the weight of mass of water in cylinder.

$P_1A + AHDg - P_2A = 0$ (3)

where P_1 is pressure at upper end and P_2 is pressure at the lower face. A is area of upper and lower surfaces.

Pressure exerted by liquid column, $P_2 - P_1 = P$ (4)

or $P = DHg$ (1)

The unit of pressure, Pascal was defined by simply putting value of H equal to 0.76m. So, Eq.(1) becomes

P=13,600kgm^{-3} ¿ 9.8ms^{-2} ¿ 0.76m=1.013 ¿ 10^5 Pa (5)

It may be regarded as standard equation for atmospheric pressure. In eq.(1) H is height of imaginary cylinder of liquid which is submerged in the same liquid (the same liquid is above and below the cylinder).Whereas in case of mercury barometer , the height H of liquid column is 0.76m , above the mercury column there is vacuum . Whereas while deriving P =DgH, the same liquid is considered above and below the imaginary cylinder of liquid.

Further according to Newton's law of gravitation, the acceleration due to gravity decreases as altitude increases. The acceleration due to gravity at the surface of the earth

$g = GM/R^2$ (6)

where G is gravitational constant, M is mass of the earth and R is radius of the earth. Acceleration due to gravity g_h at height h is

$g_h = GM/(R+h)^2$ (7)

From eq.(6) and eq.(7)

$g_h = g/(1+2h/R)^2$ (8)

The acceleration due to gravity at height h is given by eqs.(7,8) and acceleration due to gravity by method of variation of pressure is given by

$g_H = P/DH$ (9)

Both eqs.(8,9) give value of g , but by entirely different methods.

2.0 Comparison for acceleration due to gravity by both equations i.e. $g_H = P/DH$ and $g_h = g/(1+h/R)^2$

Now at different heights acceleration due to gravity by both equations i.e. $g_h = P/DH$ and $g_h = g/(1+h/R)^2$. In one equation g_h depends upon P, D and H whereas in other it only depends upon altitude, h. According to $g_h = g/(1+h/R)^2$ the value of g decreases steadily whereas according to $g_H = P/DH$, the same decreases abruptly.

The measurement of g due to ‘method atmospheric pressure’ and ‘variation with altitude’, implies that g decreases with height, h. According to eq.(8) g depends upon height h. Whereas in eq.(1) or eq.(9) g indirectly depends upon height h as pressure decreases causing decrease in height of liquid column H. At height of 50km the total air is only 1% implying considerable decrease in pressure as atmospheric pressure decreases. At height of 50 km, atmospheric pressure is 75.944 Pa and the same at height of 25 km is 2511.02 Pa. Thus accordingly g will decrease.

(i) The variables in eq.(1) are density of liquid which can be kept constant, pressure P can be measured by various methods , height of the column H has to be directly measured. There are no such factors like diameter of tube (tubes can be of different diameters), viscosity, surface tension, capillarity and angles of contact etc. Eq.(1) is valid for all liquids and tubes. The various characteristics of liquids, are shown in Table I.

(a) For mercury barometer, the height of liquid column is 0.76m. It is clear from eq.(5) the height of mercury column is regarded as standard and give unit of pressure 1.013 ¿ 10^5 Pa. This magnitude of pressure may be used for calculating the height of liquid columns of various liquids.

(b) The heights of liquid columns of ethyl alcohol, water and glycerine barometers are given by 13.16m, 10.33m, and 8.2m at sea level. In spite of availability of precise measurement techniques such experiments are not conducted yet.

(i)The acceleration due to gravity also varies with altitude, and theoretically tends to zero at infinity, according to equation $g_h = g / (1+h/R)^2$. Also according to $g_H = P/DH$, acceleration due to gravity becomes zero when P=0 provided D and H are finite. At higher altitude the pressure decreases abruptly, hence g_H.

(ii) The various typical predictions for heights of liquid columns are shown at Table II. At distance of 2km the heights of mercury, water and glycerine columns are 0.5969m, 8.1158m, and 6.4411m. These values are lower than values at sea level and higher than values at 8km (typical height of peaks of Mount Everest. For completeness the values H are determined at heights 25km (2511.02 Pa) and 50km (75.944 Pa) above sea level.

(c) Acceleration due to gravity is more at poles than at equator, hence accordingly the height of liquid column will vary accordingly.

3.0 Critical discussion

(i) According to equation P = DgH, the pressure is completely independent of other factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity etc.

There is no factor which takes in account the diameter of the tube in which height of liquid column is measured. Theoretically, the height of liquid column must be same for capillary tube (closed upper end) and tube of diameter two feet. However the phenomena of rise or fall of liquids is observed in capillary, whereas upper end is open. This aspect is not taken in account in equation P =DgH.

(ii) Mercury does not wet the walls of container, whereas water does. This property is likely to affect the height of water column.

(iii) Similarly glycerine is the most viscous liquid the viscosity of glycerine is 689.67 times that of mercury and 1058 times that of water. It causes internal resistance of the fluid and retards motion; it may affect the stabilization of liquid column. The viscous fluids also wet the tube.

(vi) The acceleration due to gravity at any height ($g_H = P/DH$) can be calculated by measuring atmospheric pressure P and height of liquid column, H. At any height the atmospheric pressure P, can be known from standard atmospheric calculator and H has to be experimentally confirmed for various liquids (mercury, water, glycerine etc). Thus g_H can be measured.

On the other hand acceleration due to gravity $g_h = g/ (1+h/R)^2$, varies with height which can be easily estimated. Now both values of g must be the same. For consistency of both values of g , the heights of liquid columns are shown in Table II.

(v) Thus undoubtedly mercury may be regarded as ideal liquid for such measurements as it has low viscosity hence comes to rest quickly, and does not wets the walls of glass tube. But the experiments has to be conducted under diverse conditions for various liquids, in view of eq.(1) for general conclusions.

Table 1: Comparison of various characteristics of Ethyl alcohol, water, glycerine and mercury.

Characteristic	**Ethyl alcohol**	**Water**	**Glycerine**	**Mercury**
Density(**kg/m^3)**	**785**	1,000	1260	13600
Coeff. Of viscosity (poise)	1.32 ¿ 10^{-4}	1.01 ¿ 10^{-2}	10.69	15.5 ¿ 10^{-3}
Surface tension(dyne/cm)	**22.3**	75.6	63.1	465 (dyne/cm)
Angle of contact	<90	8-9^0	--	137
Capillarity	Rise	Rise	--	Fall
Physical behaviour	Wets	Wets	Wets	Does not Wet
Height of liquid column (h) in m	13.16	10.33	8.202	0.76

Table II Heights of liquid columns of different liquids in tubes of different diameters, at different heights.

Sr.	Height	Pressure	Acceleration	Mercury	Water (H_1)	Glycerine (H_2)

No	(m)		(g)	H=P/Dg	H_1=P/D_1g	H_2=P/D_2g
1	0	101300	9.8	0.76	10.33	8.202
2	1	89,874.57	9.7969	0.6743	9.1737	7.2807
3	2	79,485.2	9.7938	0.5967	8.1158	6.4411
4	3	70,108.5	9.7908	0.5265	7.1606	5.683
5	4	61,640.2	9.7877	0.4630	6.2977	4.9981
6	5	54,019.9	9.7847	0.4059	5.5208	4.3816
7	6	47,181.	9.7816	0.3546	4.8234	3.8281
8	7	41,060.74	9.7785	0.3087	4.1990	3.3326
9	8	35,599.8	9.7755	0.2677	3.6417	2.8902
10	25	2511.02	9.6486	0.01915	0.26047	0.20672
11	50	75.944	9.50078	0.00058	0.007999	0.006252

4.0 Conclusions

The acceleration due to gravity, g varies with altitude and it can be measured with two methods i.e. g_H = P/DH and $g_h = g/(1+h/R)^2$. The values of g_h by both methods must be the same. The acceleration due to gravity has not yet been measured by former method. Moreover in this case there are many parameters for which equation to be tested, only then general conclusions can be drawn. For consistency of both the methods the values of height of liquids columns (for mercury, water and glycerine,) are 0.6743m, 9.1737m and 7.2807m at height of 2km above sea level. Such observations have not been taken by scientists in the existing literature. In this regard it can be added that equation g_h = P/DH was derived for imaginary cylinder of liquid, which is submerged in the liquid of the same density. Whereas in determination of height of liquid column, the above the liquid column there is vacuum. Further apparently some significant factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity etc have been excluded. Also the equation was derived for imaginary cylinder in equilibrium with liquid at rest. However it is applied for practical purposes as cited above. Such factors may affect the results. Thus sensitive experiments are required to compare the both values of g. It is equally possible in view of diverse parameters of various liquids, mercury has to be regarded as standard in measurements of pressure. The value of g due to altitude decreases steadily, whereas due to atmospheric pressure g decreases abruptly. So sensitive experiments are absolutely necessary to draw concrete conclusions.

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