the general science journal

# A Note on the Lanczos Potential for the Kerr Metric

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**Abstract.**- We show that, if we change the sign of two strategic components of the metric tensor in Kerr geometry, then we obtain a generator for the Lanczos potential of this rotating black hole.

**Key words:** Lanczos potential; Kerr metric.

PACS Nus: 04.20.-q ; 04.90.+e

Lanczos [1] showed, for any spacetime, the existence of a potential $K_{abc}$ with the properties:

$$K_{abc} = -K_{bac} \quad , \quad K_{abc} + K_{bca} + K_{cab} = 0 \quad , \quad \textbf{(1)}$$

which generates the conformal tensor via the relation [2-8]:

$$\begin{aligned} C_{abcd} &= K_{abc;d} - K_{abd;c} + K_{cda;b} - K_{cdb;a} + \\ &+ \frac{1}{2}\left[ g_{ad}\left(K_{bc} + K_{cb}\right) - g_{ac}\left(K_{bd} + K_{db}\right) + g_{bc}\left(K_{ad} + K_{da}\right) - g_{bd}\left(K_{ac} + K_{ca}\right)\right] + \\ &+ \frac{2}{3}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)K^{pq}{}_{p;q} \quad , \end{aligned} \quad \textbf{(2)}$$

such that

$$K_{ij} \equiv K_i{}^p{}_{j;p} - K_i{}^p{}_{p;j} \quad . \quad \textbf{(3)}$$

Given the Weyl tensor, it may be a formidable task to construct a Lanczos generator by integrating directly the system (2) of differential equations for the unknown $K_{abc}$. However, here we exhibit that the Lanczos potential, for the important geometry of Kerr [9], can be generated with remarkable simplicity. In fact, we consider the metric of a rotating black hole [10] in Boyer-Lindquist [11] coordinates $(r, \theta, \vartheta, t)$ with signature +2:

$$ds^2 = \frac{\Sigma}{C}dr^2 + \Sigma d\theta^2 - \frac{4amr\,Sin^2\theta}{\Sigma}d\vartheta dt + Sin^2\theta\left(r^2 + a^2 + \frac{2a^2mr\,Sin^2\theta}{\Sigma}\right)d\vartheta^2 - \left(1 - \frac{2mr}{\Sigma}\right)dt^2 \quad , \quad \textbf{(4)}$$

where

$$\sum = r^2 + a^2 Cos^2\theta \quad , \quad C = r^2 - 2mr + a^2 \quad , \quad \textbf{(5)}$$

and also the following symmetric tensor:

$$\left(S_{ij}\right) = \begin{pmatrix} \in g_{11} & 0 & 0 & 0 \\ 0 & \in g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & g_{34} \\ 0 & 0 & g_{34} & g_{44} \end{pmatrix} \quad , \quad \textbf{(6)}$$

being $g_{ab}$ the metric tensor corresponding to (4). If in (6) we put $\in = 1$ then $S_{ij} = g_{ij}$ with the known property $g_{ab;c} = 0$. But if we take $\in = -1$ then from (6) we can find the solution to (2) by constructing the Lanczos potential for the Kerr metric via the expression:

$$K_{ijk} = \frac{1}{4}\left(S_{ij;k} - S_{ik;j}\right) \quad . \quad \textbf{(7)}$$

It is a true surprise that a simple change of sign in $\in$ gives us a solution for the complicated system (2): An open problem is to find the underlying principle in this result for the black hole with rotation.

The authors are thankful with Prof. Louis Witten and Prof. Ezra Newman for their interest in this Note.

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