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"R_4 Embedded into E_5 with Energetic Rigidity"

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ABSTRACT. We exhibit a type *D* metric embedded into E_5 for the case of energetic rigidity.

Key words: Local and isometric embedding; Immersed Riemannian spaces.

We say that a R_4 is locally and isometrically embedded into E_5 if and only if there exists the second fundamental form tensor $b_{ac} = b_{ca}$ that satisfies the equations [1-9]:

$$R_{ijkc} = \varepsilon\left(b_{ik}b_{jc} - b_{ic}b_{jk}\right)$$ Gauss

(1)

$$b_{ij;k} = b_{ik;j}$$ Codazzi

where R_{ajqc} is the curvature tensor for R_4, $\varepsilon = \pm 1$ is the indicator of the normal to this 4-space and *;c* denotes the covariant derivative. A spacetime of class one has energetic rigidity if [10,11];

$$b_{ac} = \alpha R_{ac} - \frac{1}{6}\left(\alpha R + 2\beta\right)g_{ac} \quad , \qquad \alpha, \beta \text{ constants} \quad \textbf{(2)}$$

being $R_{ac} = R^{i}_{aci}$ the Ricci tensor and $R = R^{a}_{\ a}$ the scalar curvature.

In *ref.* [12] it was claimed that any R_4 verifying (2) must be conformally flat, that is, type O in the Petrov classification [3]; here we show that this affirmation is incorrect by exhibiting a counterexample. In fact the following metric:

$$ds^2 = \frac{1}{2\varphi^2}\left(d\theta^2 + d\varphi^2\right) - 2drdu \quad \textbf{(3)}$$

accepts embedding into E_5, and its corresponding b_{ij} satisfies (1) for $\varepsilon = -1$ and has the structure (2) with

$2\alpha = -\beta = \sqrt{2}$. However the metric (3) is type *D*!.

Finally, we express an explicit embedding of metric (3), in the sense of [1,5,13-15], given by:

$$z^1 = \frac{1}{\varphi\sqrt{2}}\cos\theta \quad , \qquad z^2 = \frac{1}{\varphi\sqrt{2}}\sin\theta \quad , \qquad z^3 = \frac{1}{\sqrt{2}}(u-r) \quad , \quad \textbf{(4)}$$

$$z^4 = \frac{1}{\sqrt{2}}\left[\ln\left(\varphi + \sqrt{\varphi^2 - 1}\right) - \frac{\sqrt{\varphi^2 - 1}}{\varphi}\right] \quad , \qquad z^5 = \frac{1}{\sqrt{2}}(u+r)$$

then (3) adopts the pseudo-Euclidean form:

$$ds^2 = (dz^1)^2 + (dz^2)^2 + (dz^3)^2 + (dz^4)^2 - (dz^5)^2 \quad \textbf{(5)}$$

Therefore the spacetime associated with (2) is algebraically special but not necessarily type *O*.

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